System Architecture: Complexities Role in Architecture Entropy

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Abstract - *This is the second paper in a series that discusses and further defines the notion of system architecture entropy. This paper specifically looks at the correlation between system architecture entropy and system complexity. The role of complexity, as it relates to entropy, is established by investigating entropy-complexity relationships in biology and information theory. Those concepts are then used to assert the relationship of complexity and entropy in system and system of systems (SoS) architecture. Along with defining the entropy-complexity system architecture relationship, examples of systems that demonstrate it are provided. Application of lessons noted during the process of establishing the system architecture entropy-complexity relationship are provided so that practitioners can immediately begin to apply the best practices that were uncovered through this study.*

Keywords: Architecture, Complexity, Entropy, Systems Engineering

1 Introduction

This manuscript is the second in what has become a series of papers that develops and defines the concept of entropy as applied to a System and System of Systems (SoS) Architecture. The initial concept of system architecture entropy was introduced in 2009 by Cloutier et al. [14]. This manuscript expands on that discussion, and further explains how system complexity contributes to system architecture entropy. The foundation will be laid to enable systems engineers to evaluate system architectures and perhaps more importantly predict future behavioral trends of the system architecture. The ability to predict the impact of system architecture entropy is of utmost importance as systems become more complex and the engineering community begins focusing not on *systems* but on *systems of systems* (SoS).

This manuscript proposes that the systems engineering community can apply the concepts of entropy and complexity in a similar manner as other disciplines. The landscape of entropy-complexity analysis has not been defined relative to systems engineering, and to begin developing that relationship and therefore expanding the architecture entropy landscape is one goal of this research.

The notion of contemplating and predicting architecture entropy along with the complexity of a system architecture is important because a system can be complex but have low entropy therefore investment (time, money, etc) in the system could be justified. At the other extreme, a system which has high entropy but is not complex may indicate that the system architecture is at the end of its life and the investment may be better utilized to develop a new architecture. Therefore complexity alone may not be the best indicator as to where organizations should or should not focus but instead they need to look at the entropy-complexity relationship.

2 Entropy and Complexity Relationship

The relationship between entropy and complexity is one that has been studied in a variety of disciplines. Those disciplines that have the most in depth studies of the entropy-complexity relationship and are most relevant to engineered systems are the disciplines of *information theory* and *biology*. Discussion of information theory and biology are important to the field of systems engineering to establish a historical and proven relationship that this paper will extend to engineered systems.

2.1 Information Theory

In the discipline of information theory, Claude Shannon's 1948 seminal paper defined entropy in terms of a communications system [5]. Shannon's Entropy demonstrated how much information in a message is useful based on the probability of receiving a message. The amount of information gain between states is inversely proportional to the logarithm of the probability of a state occurrence. Shannon's communication entropy (H) is represented as:

$$H = -K \sum_{i=1}^{n} p_i \log p_i \qquad (1)$$

where K is a positive constant, p_i is the probability of an event occurring, and log p_i is the uncertainty related to that event.

Li [12] observes that "several authors speculate that the typical relationship between complexity and entropy is a unimodal one…" The speculation that Li [12] mentions is backed by data created by Crutchfield and Young [18] and shown in Figure 1. This data shows that in the realm of information theory the entropy-complexity relationship is unimodal with complexity being small for small and large entropy values but large for intermediate entropy values, which is the same observation Li [12] made. Li goes on to point out another important characteristic of the complexity-entropy relationship which is it "depends on the specific structure of the short-range correlations (which is captured by the "grammar") [12]."

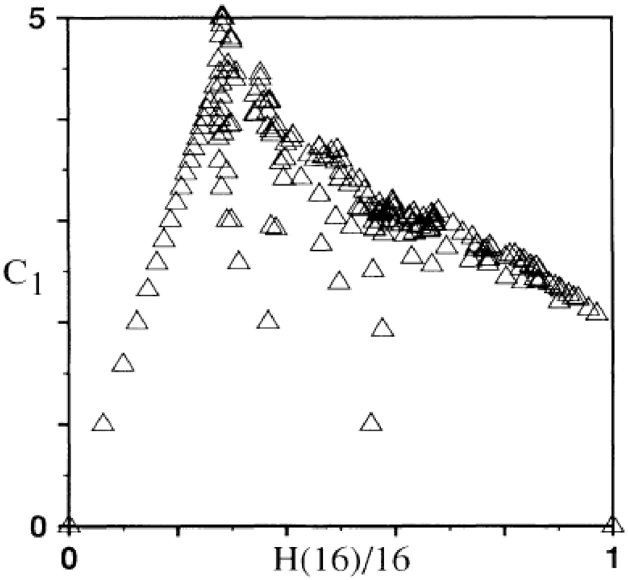

Figure 1 Complexity (C) Versus Entropy (H) in Crutchfield and Young's study [18]

The complexity-entropy relationship in information theory is also shown through Kolmogorov Complexity and Shannon Entropy. The relationship between these two topics in information theory is well established [6] [15]. The complexity-entropy relationship in information theory has been modeled mathematically, is unimodal, and is dependent on the specific structure of the short-range correlations.

2.2 Biology

While the application of entropy is still debated in biology it is a widely used term and concept to measure biological order [10]. Schrodinger [17] discusses entropy of biological systems and introduces the idea that biological systems may have negative entropy, defying the Second Law of Thermodynamics, which states entropy can only remain the same or increase in a system. This idea of negative entropy is one of the main stimuli for the debate in biology regarding the use of the term "entropy".

On the other hand, complexity is one area of study that biologists agree upon, and it is well understood that most biological systems have some level of complexity, with some types of biological systems having the highest level of complexity currently known to mankind. The idea of biological complexity is also widely studied as "evolution complexity" which deals with the complexities of how organisms adapt and/or evolve.

Biologist have also identified and studied the relationship between entropy and complexity. Brook [8] states a biological system: "*…realizes that increases in complexity and increases in organization are, themselves emergent properties of this entropic behavior.*" Roper of Virginia Polytechnic Institute and State University summarizes much of the complexity and entropy research in biology by stating, "one only has to look at nature to see low entropy and complexity work together [3]." Though this statement may simplify the association, it demonstrates the profound relationship of entropy and complexity within biological systems.

The concept of entropy in biology is utilized to understand one of its fundamental areas of study, evolution. This importance is documented in many different medias and in particular one book appropriately titled "Evolution as Entropy: Toward a unified theory of biology" [8] which has been citied at least 322 times, per www.scholargoogle.com. Brooks and Wiley [8] go on to describe the relationship between complexity and entropy: "*…as a developing cell type or organism becomes more complex, accession of new microstates affects a progressively smaller proportion of the system. Thus, the actual increase in entropy lags further behind the maximum possible entropy increase… , this increasing lag signals increased organization, while increasing entropy signals increasing complexity.*" This particular example from biology is interesting because it identifies that when a change occurs that makes the organism (system) more complex there is a maximum entropy change that could occur. The author goes on to state that in biology they actually see that the real entropy change is less than the possible maximum.

The biology discussion here helps demonstrate how complexity and entropy are related in other fields of study. The biology example also serves as an example of where entropy from information theory has been synthesized into a quantifiable measure that helps evaluate biological systems much like this research is attempting to do with engineered systems.

3 Current Systems that demonstrate Complexity-Entropy Relationship

In this section three different engineered systems will be discussed in terms of complexity and entropy. These systems are: 1) manufactured systems, 2) Power Grid System, and 3) Railway Transportation System. The Power Grid and Railway Transportation System are both Systems of Systems. These systems were chosen for their varying characteristics.

3.1 Manufacturing Systems

One example of an engineered system that demonstrates the relationship between system complexity and entropy is a manufacturing system. Deshmukh et al. [11] state the "entropy measure for static complexity is an aggregate indicator of routing, process, and product flexibilities related to a set of parts." Deshmukh et al. [11] uses Shannon's entropy measure [5] to develop the proof for static complexity for part mix which is defined as [11]:

$$Hp = -C\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{r}\sum_{l=1}^{n} \tilde{\pi}_{ijkl} \log \tilde{\pi}_{ijkl} \qquad (2)$$

One can see by comparing this proof to Shannon's communication entropy shown in Section 2.1 that Deshmukh et al. [11] has actually developed a proof for what they have termed "static complexity" in terms of "entropy measure" from information theory.

Deshmukh et al. [11] goes on to observe that there is a direct relationship between system flexibility and complexity, stating that "Static complexity measures the total number of decisions that related to a part mix and hence, can be considered as an aggregate indicator of these (system) flexibilities." This deviation in discussion from entropy and complexity to complexity and flexibility reinforces the idea that system architecture entropy has indirect influence on the performance of a system, in this case flexibility.

This section has shown that in manufacturing systems there is an established relationship between complexity and entropy. While [11] set entropy measure and static complexity equal to one another, the authors believe there is a larger set of variables necessary to fully describe the measure of entropy in any system. In its current state the research in [11] does provide relevant insight into the usefulness of complexity-entropy relationship by discussing how it affects system flexibility and the ability to predict such flexibility. This discussion of manufacturing systems also demonstrates that the relationship of entropy-complexity is applicable to engineered systems of varying complexity and varying entropy.

3.2 Power Grid System

The power grid is an example of a complex SoS which has evolved over time. It is an example of a SoS which demonstrates the problems with increased system architecture entropy. It has been stated by Peter Fox-Penner, a former senior official at the U.S. Department of Energy that; "The architecture of the grid no longer matches its role, and no one has the authority or money to change it [19]." Therefore, it is important that research and development efforts focus on upgrading the power grid system instead of fully redesigning it. In the context of "upgrading" as opposed to starting from scratch, studying the cascading effects of faults on power system functionality and adopting an approach that can predict failures is crucial [16].

While the assumed increase in system architecture entropy in the power grid is debatable, [1] has developed the idea of "power flow entropy". Power flow entropy is an example of documented entropy which resembles that which is being discussed in this manuscript as system architecture entropy. Bao [1] develops power flow entropy "to quantify the overall heterogeneity of load distribution" on the power grid system. The goal of power flow entropy is in line with the goal of extending system architecture entropy to include a quantification of the overall heterogeneity of engineered systems. In this context, heterogeneity of the system can be seen as synonymous to complexity. Bao [1] has directly linked complexity (heterogeneity) of the power grid system to entropy of the power grid system.

One of Bao's [1] key observations is that "the relationships (between power flow entropy and power failure) are reasonable because the sharp increase of flow entropy implies the increasingly intensifying heterogeneity (complexity) in the load distribution, which certainly can speed up the cascade, and the mitigation of failure spreading suggests the gradual homogeneity of power flow distribution." This observation is important as it is one example of a case where a specific type of system architecture entropy ("power flow entropy") can be used to predict the behavior of a system, in this case power failure in the power grid.

The one shortfall of Bao's [1] analysis of power flow entropy is that it only considers factors within the power grid system. Bao [1] defines the power flow entropy as follows:

$$H(t) = -\sum_{i=1}^{M} \frac{n(i)}{N} \log \frac{n(i)}{N} \qquad (3)$$

where, n(i) is the number of lines, whose load/capacity ratio fall within the nth interval. In other words, Bao [1] only considers the line load, the line capacity, and the number of lines when calculating the power flow entropy. This may be an overly simplistic view of power flow entropy. Power grids are decentralized (no single organization supervises a large power grid), highly complex systems comprised of power generators, transmission lines, and power consumers. In relationship to the entire SoS, Bao [1] fails to consider other factors that may influence the power flow entropy such as generation or end point distribution issues, multiple organization forces, security (level of exposure to malicious attacks), and supply chain management. By being able to assess the entropy of the power grid system, taking all of the necessary factors into consideration, it may be possible to predict the performance of the system as a whole, and the various subsystems of the power grid. This would improve the predictability of the system performance and reduce the potential exposure to cascading failures or blackouts.

The discussion of the power grid and Bao's [1] power flow entropy has again demonstrated that others, have begun to describe the relationship between system complexity and system entropy. This example also demonstrated a system that has high system architecture entropy along with high architecture complexity.

3.3 Railway Transportation System

The railway transportation is a rather simple or low complexity system of systems. The railway transportation SoS has been around for over a century, and while the systems within the overall system have became more complex the SoS has remained almost unchanged. One may argue that it is an architecture of medium complexity. Although the SoS is rather low complexity, the related system architecture entropy is rather high. This high level of entropy is outwardly apparent through the architecture's inflexibility and scaling issues.

The author's current theory of why the architectural entropy of a railway transportation system is high is due to the differential of architectural complexity within the systems of the SoS. Some of the lower level systems such as the track architecture have low complexity while other systems such as the locomotives and signaling equipment have higher (and increasing) complexity.

While there does not appear to be any published research demonstrating the entropy-complexity relationship of a railway transportation system it is presented here to round out the examples and provide an example of a system of systems that has low-medium complexity but high entropy.

4 System and SoS Architecture Complexity and Entropy

From Section 2, there appears to be an emerging pattern in which once entropy and complexity are defined in a particular field of study, a relationship between the two can then be established. This section will use the concepts from the previous sections to convey to systems engineers that there is indeed a relationship between entropy and complexity in systems and SoS architecture. This section will begin to discuss the relationship of entropy and complexity as it appears in the architecture of engineered systems. While system architecture entropy is a newly explored concept in systems engineering [14], complexity of engineered systems has been researched and documented for some time now [13]. In preparation for a discussion of the entropy-complexity relationship this section briefly discusses system architecture entropy and system complexity. It is important to note that currently system complexity and system *architecture* complexity are, for the most part, treated as equivalent in the systems engineering literature and are not specifically distinguished, though it is the belief of these authors that there is a distinction that should be made in the future.

4.1 System Architecture Entropy

The current working definition for system architecture entropy is: *a measure of disorder in the system architecture that grows more disordered over time as the architecture evolves to satisfy new requirements [14]*. Currently the authors hypothesize that as a system or SoS evolves the level of architecture entropy increases. Examples of increasing entropy as systems evolve are discussed in [14]. The examples include Naval Ships, Automobiles, and Microsoft Windows.

4.2 Engineered System Complexity

The understanding of system complexity dates back to 1948 [13]. The two forms of system complexity defined by [13] are organized and disorganized. Organized complexity is a system that has a "sizable number of factors which are interrelated into an organic whole." Disorganized complexity is just the opposite and is a system that "becomes unmanageable, not because there is any theoretical difficulty, but just because the actual labor of dealing in specific detail with so many variables turns out to be impracticable." This idea of complexity is widely recognized in engineered systems and one can quickly find many articles and research groups such as the Institute of Complex Engineered Systems (an agent within Carnegie Mellon University) that are dedicated to its study.

It is important to understand that at the present "system" in much of the literature refers to all forms of systems, including biological, engineered, social, system of systems, etc. Therefore, "system complexity" is still used much of the time as an all-encompassing term.

4.3 System Architecture Entropy and Complexity Relationship

This research endorses that system architecture complexity and system architecture entropy are two separate and quantifiable characteristics of a system. Although the two characteristics can be identified separately, this research also hypothesizes that there is a definable relationship between the two. Currently, the work has not progressed to the point of thoroughly describing how the entropy-complexity can be expressed; it is able to confidently state there is a relationship as demonstrated in the three examples discussed in this manuscript. At present, the relationship does not appear linear (Table 1).

Table 1 - Nonlinear System Complexity-Entropy Relationships

System	Complexity	Entropy
Railway transportation SoS	Low to medium	High
Power grid	High	High

The non-linear relationship is not a surprise since as Figure 1 displays a unimodal relationship was defined for complexity versus entropy in information theory. While this discovery of a non-linear relationship may seem trivial it is actually of considerable use to practitioners. It serves as a cautionary advisory to those developing or upgrading systems that while a system may be low in complexity it could be high in entropy, indicating a system architecture that may need to be retired or fully re-engineered.

A major driver of complexity is technological advancement. For systems in rapidly developing domains, often the next best component is developed prior to the system's fielding. When the system is in the field, upgrading to the more advanced component may cause interface issues that can be avoided through rigorous adherence to interface standards. In this way, reducing the effect of technology turnover on complexity will in turn act to keep architecture entropy low.

A system that has tightly coupled components enables their complete removal and replacement as the system evolves. This allows for outdated components and unwanted redundancy to be easily removed from a system, therefore reducing overall architectural entropy.

As a system evolves, there is a tendency to make changes as needed with little documentation. Without an accurate image of the architecture, when legacy systems are developed, architects make decisions that may cause additional complexity. Proper documentation or systems modeling allows developers to make proper architectural decisions that will keep complexity and architectural entropy low.

5 Applications from Complexity-Entropy Relationship

System architecture entropy as a whole has not been quantified, nor are there algorithms yet to sufficiently model this research. However, the research has wielded useful lessons that can be applied to systems engineering today.

The first lesson is that open systems (versus closed systems) seem to exhibit lower levels of entropy. Therefore if systems engineers develop systems that have open interfaces then the system architecture entropy is likely to be lower [3].

The second lesson is that high system modularity or systems that can be broken into smaller pieces will likely have lower entropy even though they may be highly complex. These findings point to the fact that system architectures should be developed so that they can be broken into smaller sub-systems or components with ease to keep system architecture entropy low. From this research it is clear that those smaller pieces should be architected in such a manner as to minimize interfaces. Further research should be conducted on the payoff between modularity and number of interfaces with regards to impact on system architecture entropy.

The third lesson that can be applied to engineered systems from the study of complexity-entropy relationship comes from biology. The goal should be to engineer low entropy systems no matter what the complexity. From biology one can see that good examples of consistently low entropy systems are those that are self organizing. Therefore even before architecture entropy is fully defined it can be speculated with confidence that engineered systems that have more characteristics of a self organizing system will have lower entropy than those that have fewer self organizing characteristics. Characteristics of self-organizing systems that have been identified by [2] are: 1) The various systems or subsystems must be coupled with one another so they can interact, 2) This interaction must be self-sustaining, or autocatalytic, and 3) The self-organizing system must produce functions that are useful to the system's stakeholders. This particular lesson has been identified as beneficial to mostly software systems [2].

6 Future Research

Just as biologist must continue to search for answers to how complex organisms have evolved, systems engineers must begin to understand how complex engineered systems evolve. To understand the future of a complex system architecture one must first understand the systems architectural entropy. While the relationship between, only system architecture entropy and system complexity was addressed, the authors are firmly convinced (both from personal experience and initial research) that many other factors (see Figure 2) also contribute to the entropy of a system's architecture. This list of factors is not extensive and it is the author's belief that as this research progresses, the list of factors contributing to the entropy of a system's architecture will itself evolve and change. The seven variables in Figure 2 will also be researched and their role in architecture entropy will be further developed, verified, and quantified.

In the future this research will develop a generalized engineered system architecture entropy definition but until then practitioners can use Bao's [1] research as an example to uncover the entropy-complexity relationship for their field of study. The motivation for others to follow Bao's lead would be for the same reason Bao found and that is, system predictability.

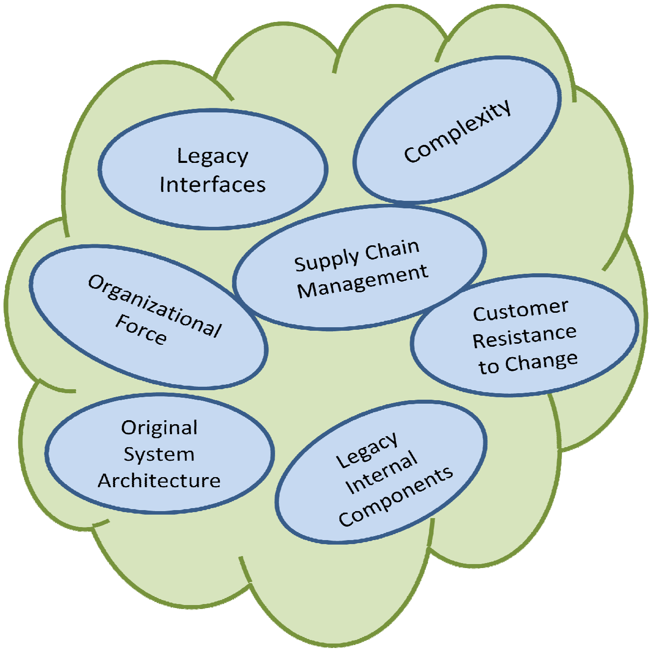

Figure 2 System Architecture Entropy Variables

The authors also intend on continuing research on this topic to expand the understanding of the impact system architecture entropy has on cost. Other research questions to be investigated include: 1) what role does architecture management play in minimizing the impact of architecture entropy, 2) once the impact and cost of system architecture is better understood, how can systems engineers cope with architecture entropy, and 3) how can system architecture entropy be used to predict system architecture performance.

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